



# 12

## NUMBER AND ALGEBRA

# GRAPHING LINEAR EQUATIONS

The famous French philosopher and mathematician René Descartes (1596–1650) is famous for discovering the connections between algebra and geometry. His concept of coordinate geometry connected algebra and geometry using the graphs of straight lines and curves. In computer modelling, the use of coordinates allows us to locate things more precisely, especially in three-dimensional (3D) space. Number plane graphs are used to track changes over short and long periods of time, such as the height and speed of an aeroplane.



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## Chapter outline

	Working mathematically				
12.01	Tables of values	U	F		
12.02	Finding the rule	U	F	R	C
12.03	Finding rules for number patterns	U	F	PS	R C
12.04	The number plane	U	F	R	C
12.05	Graphing number patterns	U	F	R	C
12.06	Graphing linear equations	U	F	R	C
12.07	Finding the equation of a line	U	F	R	C
12.08	Comparing linear equations	U	F	R	C
12.09	Solving linear equations graphically	U	F		
12.10	Intersecting lines	U	F	R	

## Wordbank

**consecutive numbers** Numbers that follow each other in order, for example, 2, 3, 4

**constant term** The term in an equation that is a number only and does not contain a variable, for example, the 5 in  $y = 2x + 5$

**horizontal** Going across, sideways, flat

**linear** Involving a line

**linear equation** A formula whose graph is a straight line

**number plane** A coordinate grid system based on the  $x$ - and  $y$ -axes, also called a Cartesian plane

**table of values** A table of ordered pairs of numbers, usually following a formula and which can be graphed on a number plane

**vertical** Going up and down, at a right angle to the horizontal

**$y$ -intercept** The  $y$ -value at which a line crosses the  $y$ -axis on the number plane

## In this chapter you will:

- complete a table of values for a linear equation and graph the linear equation on a number plane
- identify and describe a number pattern, then represent it algebraically and graphically on a number plane
- determine the formula for a table of values and set of points on a number plane
- compare and contrast the graphs of different linear equations
- graph simple non-linear equations
- solve linear equations graphically
- graph 2 intersecting lines and identify the point of intersection

## SkillCheck ANSWERS ON P. 581

1 For each equation, find the missing number.

**a**  $2 \times 3 + \underline{\quad} = 10$

**b**  $2 \times (-1) + \underline{\quad} = 3$

**c**  $3 \times 1 - \underline{\quad} = 1$

**d**  $-2 \times 2 + \underline{\quad} = -2$

**e**  $4 \times 3 - \underline{\quad} = 8$

**f**  $-1 \times (-1) - \underline{\quad} = -4$

2 Evaluate each expression.

**a**  $3x - 2$  if  $x = 2$

**b**  $\frac{1}{2}x + 3$  if  $x = 6$

**c**  $2x - 1$  if  $x = -1$

**d**  $-x + 1$  if  $x = 0$

**e**  $-2x - 1$  if  $x = 1$

**f**  $10 - x$  if  $x = 3$

3 Solve each equation.

**a**  $3x - 2 = 7$

**b**  $-2x - 1 = 3$

**c**  $-x + 1 = 4$

**d**  $2x - 1 = -5$

**e**  $3x + 2 = x - 1$

**f**  $x - 2 = 4 - x$

## 12.01 Tables of values



Tables of values

In order to graph straight lines, we need points. In Year 7, you were given a **table of values** to plot on the number plane, but where do the values come from? We will now use a rule or formula to create a table of values.

### Example 1

Complete this table of values using the formula  $v = 2u + 1$ .

<b>u</b>	-2	-1	0	1	2	6	9
<b>v</b>							

### Solution

Substitute each value of  $u$  in the table into the formula  $v = 2u + 1$  to find the value of  $v$ .

- When  $u = -2$ ,  $v = 2 \times (-2) + 1 = -3$
- When  $u = -1$ ,  $v = 2 \times (-1) + 1 = -1$

- When  $u = 0, v = 2 \times 0 + 1 = 1$
- When  $u = 1, v = 2 \times 1 + 1 = 3$
- When  $u = 2, v = 2 \times 2 + 1 = 5$
- When  $u = 6, v = 2 \times 6 + 1 = 13$
- When  $u = 9, v = 2 \times 9 + 1 = 19$

Completing the table, we have:

$u$	-2	-1	0	1	2	6	9
$v$	-3	-1	1	3	5	13	19

### EXERCISE 12.01 ANSWERS ON P. 581

#### Tables of values **UF**

**1** Complete each table of values given the formula.

**a**  $y = x - 1$

$x$	3	7	-1	-8	4	5
$y$					3	

**b**  $q = 3p$

$p$	8	-3	2	5	10	-4
$q$				15		

**c**  $k = h \div 2$

$h$	12	8	-2	-6	0	7
$k$	6					

**d**  $y = x + 7$

$x$	-2	11	7	0	13	-9
$y$		18				

**e**  $b = 2a$

$a$	8	-1	-5	0	4	2
$b$			-10			

**f**  $y = x - 3$

$x$	10	-4	7	5	-8	11
$y$	7					

**2** Given the rule  $y = -x - 4$ , what is the value of  $y$  when  $x = -3$ ? Select the correct answer **A, B, C** or **D**.

**A** -1

**B** -7

**C** 7

**D** 12

**3** Complete each table of values given the formula.

**a**  $p = 5m - 1$

$m$	3	10	1	4	8	-6
$p$		49				

**b**  $q = 3p - 2$

$p$	4	-1	7	10	2	-6
$q$			19			

**c**  $t = 4r + 6$

$r$	2	0	-3	9	-5	4
$t$	14					

**d**  $z = 5y + 2$

$y$	4	8	7	-5	6	-2
$z$		42				

**e**  $y = 2x - 3$

$x$	0	1	2	3	4	5
$y$						

**f**  $b = \frac{1}{2}a - 2$

$a$	-2	-1	0	1	2	3
$b$						

12.01

EXAMPLE  
1



**g**  $k = 3h + 5$

<b>h</b>	-1	0	1	2	3	4
<b>k</b>						

**h**  $y = 1 - x$

<b>x</b>	-2	-1	0	1	2	3
<b>y</b>						

**4** Which table of values matches the formula  $y = 3x - 1$ ? Select **A**, **B**, **C** or **D**.

**A**

<b>x</b>	1	2	3	4	5
<b>y</b>	2	3	4	5	6

**B**

<b>x</b>	1	2	3	4	5
<b>y</b>	2	5	8	11	14

**C**

<b>x</b>	1	2	3	4	5
<b>y</b>	4	7	10	13	16

**D**

<b>x</b>	1	2	3	4	5
<b>y</b>	0	3	6	9	12

**5** Which table of values follows the rule  $c = \frac{h+2}{2}$ ? Select **A**, **B**, **C** or **D**.

**A**

<b>h</b>	0	2	4
<b>c</b>	1	3	5

**B**

<b>h</b>	0	2	4
<b>c</b>	1	2	5

**C**

<b>h</b>	0	2	4
<b>c</b>	0	1	3

**D**

<b>h</b>	0	2	4
<b>c</b>	1	2	3

## 12.02 Finding the rule

We can often find the formula for a table of values. If the values in the top row are **consecutive** (increase by 1 each time), look for a pattern in the values in the bottom row.



Finding the rule

### Example 2

Find the formula for each table of values.

**a**

<b>p</b>	2	3	4	6	7	8
<b>q</b>	1	2	3	5	6	7

**b**

<b>m</b>	-2	-1	0	1	2	3
<b>n</b>	-6	-3	0	3	6	9

**c**

<b>r</b>	1	2	3	4	5	6
<b>t</b>	1	3	5	7	9	11

**d**

<b>d</b>	1	2	3	4	5	6
<b>e</b>	8	12	16	20	24	28

## Solution

- a What has been done to each value in the top row to get the value in the bottom row?

$p$	2	3	4	6	7	8
$q$	1	2	3	5	6	7

The pattern is:

$$2 - 1 = 1$$

$$3 - 1 = 2$$

$4 - 1 = 3$ , and so on.

The formula is  $q = p - 1$ .

b

$m$	-2	-1	0	1	2	3
$n$	-6	-3	0	3	6	9

The pattern is:

$$3 \times (-2) = -6$$

$$3 \times (-1) = -3$$

$3 \times 0 = 0$ , and so on.

The formula is  $n = 3m$ .

- c The formula for this table of values is not so obvious. It involves 2 operations: a multiplication and either an addition or a subtraction.

$r$	1	2	3	4	5	6
$t$	1	3	5	7	9	11

If the values in the top row are consecutive, the bottom row helps us find the **multiplier**.

The values in the bottom row go up by 2 each time, so the multiplier is 2. This means the formula must have  $2 \times r$  in it.

$$2 \times 1 - 1 = 1$$

$$2 \times 2 - 1 = 3$$

$2 \times 3 - 1 = 5$ , and so on.

The formula is  $t = 2r - 1$ .

d

$d$	1	2	3	4	5	6
$e$	8	12	16	20	24	28

The values in the top row are **consecutive**, the values in the bottom row go up by 4, so the multiplier is 4, and the equation is of the form  $e = 4d$  \_\_\_\_.

Choose any ordered pair from the table to find the missing number, say (2, 12).

When  $d = 2$ ,  $e = 4 \times 2 + 4 = 12$ , so the missing number is 4.

So the equation is  $e = 4d + 4$ .

(Checking that this is also true for another ordered pair (1, 8).

When  $d = 1$ ,  $e = 4 \times 1 + 4 = 8$ )

## Finding the rule **UFRC**

EXAMPLE  
2

**1** Copy and complete the formula for each table of values. **R C**

**a**  $y = \underline{\quad} x$

$x$	0	1	2	3
$y$	0	3	6	9

**b**  $y = \underline{\quad} x$

$x$	0	1	2	3
$y$	0	-2	-4	-6

**c**  $y = x + \underline{\quad}$

$x$	-1	0	1	2
$y$	3	4	5	6

**d**  $y = x - \underline{\quad}$

$x$	0	1	2	3
$y$	-4	-3	-2	-1

**e**  $y = 10 - \underline{\quad} x$

$x$	-1	0	1	2
$y$	11	10	9	8

**f**  $y = x \div \underline{\quad}$

$x$	0	1	2	3
$y$	0	$\frac{1}{2}$	1	$1\frac{1}{2}$

**g**  $k = \underline{\quad}$

$p$	1	2	3	4
$k$	5	6	7	8

**h**  $y = \underline{\quad}$

$x$	-1	0	1	2
$y$	-4	-3	-2	-1

**i**  $j = \underline{\quad}$

$h$	4	5	9	11
$j$	8	10	18	22

**j**  $p = \underline{\quad}$

$m$	0	5	30	45
$p$	0	1	6	9

**2** Find the formula for each table, then complete the last 2 columns. **R C**

**a**

$f$	1	2	3	4	7	8
$h$	1	4	7	10		

**b**

$m$	1	2	3	4	6	9
$p$	2	7	12	17		

**c**

$m$	0	1	2	3	5	8
$b$	3	6	9	12		

**d**

$h$	3	4	5	6	9	11
$k$	8	10	12	14		

**e**

$r$	0	1	2	3	4	7
$s$	1	4	7	10		

**f**

$a$	2	3	4	5	10	11
$b$	2	4	6	8		

**g**

$m$	0	1	2	3	5	6
$n$	5	8	11	14		

**h**

$c$	3	4	5	6	9	10
$d$	31	41	51	61		

**i**

$w$	3	4	5	6	7	8
$x$	14	19	24	29		

**j**

$y$	5	6	7	8	9	10
$z$	4	6	8	10		



**k**

<b>a</b>	1	2	3	4	7	9
<b>m</b>	5	9	13	17		

**l**

<b>z</b>	4	5	6	7	9	12
<b>t</b>	1	3	5	7		

- 3** What is the formula for this table? Select the correct answer **A**, **B**, **C** or **D**.

<b>p</b>	3	4	5	6
<b>m</b>	8	11	14	17

- A**  $m = 2p + 1$       **B**  $m = 3p - 1$       **C**  $m = 2p + 4$       **D**  $m = 3p + 1$

- 4** What is the formula for this table? Select **A**, **B**, **C** or **D**.

<b>x</b>	1	2	3	4	5
<b>y</b>	-5	-7	-9	-11	-13

- A**  $y = -2x - 5$       **B**  $y = 2x - 5$       **C**  $y = -2x - 3$       **D**  $y = 2x - 3$

- 5** Find the formula for each table, then complete the table. **R** **C**

**a**

<b>x</b>	0	1	2	3	9
<b>y</b>	-1	-3	-5	-7	

**b**

<b>A</b>	0	1	2	3	10
<b>C</b>	0	0.5	1	1.5	

**c**

<b>s</b>	-1	0	1	2	7
<b>t</b>	18	13	8	3	

**d**

<b>t</b>	-1	0	1	2	5
<b>v</b>	30	20	10		

**e**

<b>x</b>	0	1	2	3	6
<b>D</b>	-0.5	-1	-1.5		

## Finding rules for number patterns

12.03

### Example 3

For this number pattern: 2, 5, 8, 11, 14, ...

- a** find the 8th term  
**b** find a formula for the  $n$ th term,  $T$ .

### Solution

- a** This number pattern is increasing by 3 each time. By continuing the pattern:  
 2, 5, 8, 11, 14, 17, 20, 23  
 8th term =  $20 + 3 = 23$



Patterns and  
rules

**b** Write the number pattern as a table of values.

Term of pattern, $n$	1	2	3	4	5
Number, $T$	2	5	8	11	14

The number pattern goes up by 3 each time, so the multiplier for the formula is 3, and the equation is of the form  $T = 3n$  \_\_\_\_.

Choose any ordered pair from the table to find the missing number, say (1, 2).

When  $n = 1$ ,  $T = 3 \times 1 - 1 = 2$ , so the missing number is  $-1$ .

$$3 \times 1 - 1 = 2$$

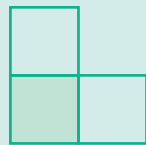
$$3 \times 2 - 1 = 5$$

$$3 \times 3 - 1 = 8, \text{ and so on}$$

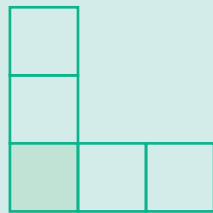
So the general formula for the  $n$ th term is  $T = 3n - 1$ .

## Example 4

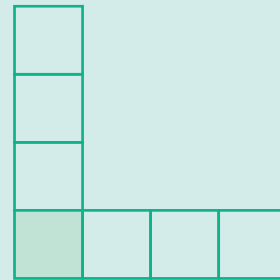
Examine this geometrical pattern.



arm length = 1



arm length = 2



arm length = 3

**a** Copy and complete this table for the pattern.

Arm length, $a$	1	2	3	4	5	6
Number of tiles, $t$						

**b** Write the rule for this pattern in words.

**c** Write the rule as a formula.

**d** How many tiles would be needed to make a design with an arm length of:

**i** 18?

**ii** 60?

## Solution

Arm length, $a$	1	2	3	4	5	6
Number of tiles, $t$	3	5	7	9	11	13

**b** The number of tiles increases by 2 each time. This means that the multiplier is 2, and the equation is of the form  $t = 2a$  \_\_\_\_.

Choose (1, 3) from the table to find the missing number.

$$\text{When } a = 1, t = 2 \times 1 + 1 = 3.$$

In words, the rule is: The number of tiles is 2 times the arm length plus 1.

- c** As a formula, the rule is  $t = 2a + 1$ .
- d i** When  $a = 18$ ,  $t = 2 \times 18 + 1 = 37$   
37 tiles are needed for a design with an arm length of 18.
- ii** When  $a = 60$ ,  $t = 2 \times 60 + 1 = 121$   
121 tiles are needed for a design with an arm length of 60.

**EXERCISE 12.03** ANSWERS ON P. 581

**Finding rules for number patterns U F P S R C**

- 1** Find the formula for the  $n$ th term,  $T$ , of the number pattern  $-1, -3, -5, -7, \dots$   
Select the correct answer **A, B, C** or **D**. **R C**

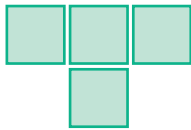
**A**  $T = n - 2$       **B**  $T = 1 - 2n$       **C**  $T = 2n - 1$       **D**  $T = n - 3$

- 2** For each number pattern: **R C**

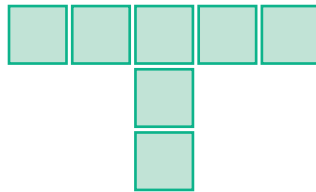
- i** find the 8th term  
**ii** find a formula for the  $n$ th term,  $T$   
**iii** use the formula to find the 8th term and the 20th term.

- a** 1, 3, 5, 7, 9, ...      **b** 2, 4, 6, 8, 10, ...  
**c** 8, 10, 12, 14, 16, ...      **d** 7, 10, 13, 16, 19, ...  
**e** 7, 15, 23, 31, 39, ...      **f** 2, 7, 12, 17, 22, ...  
**g** 19, 17, 15, 13, 11, ...      **h** 1, 4, 9, 16, 25, ...

- 3 a** Here are the first 2 T-shapes in a geometrical pattern. Draw the next 3 T-shapes. **R C**



arm length = 1



arm length = 2

- b** Copy and complete the table.

Arm length, $a$	1	2	3	4	5	8	11
Number of tiles, $t$	4						

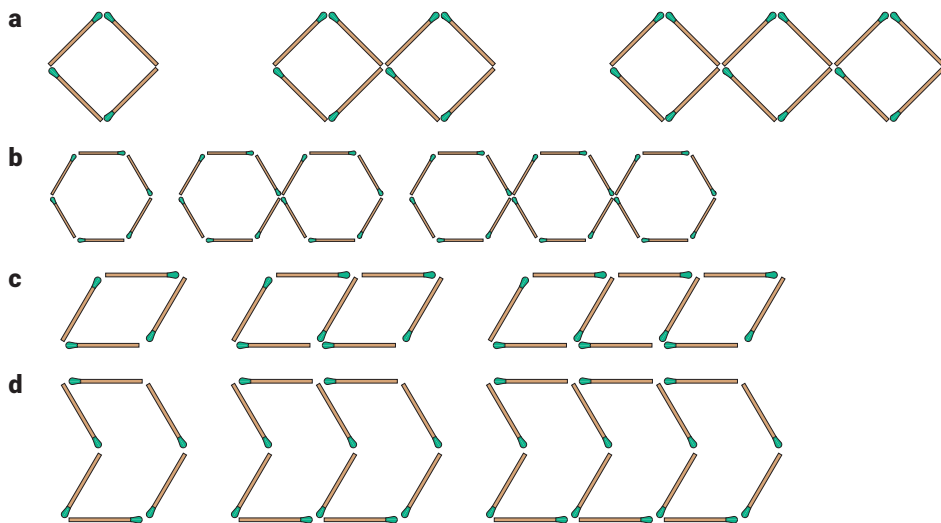
- c** Write the rule for the pattern in words.  
**d** Write the rule as a formula.  
**e** How many tiles are needed to build a T-shape with arm length of:  
**i** 15?      **ii** 33?

EXAMPLE  
3

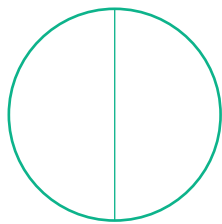
EXAMPLE  
4

12.03

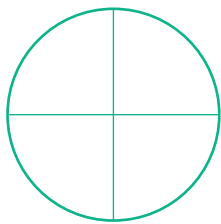




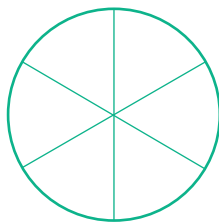
- 7** The pattern below is formed by cutting a cake  $c$  times to make  $p$  pieces. Which of the following is the formula that best describes the pattern? Select **A**, **B**, **C** or **D**. **R C**



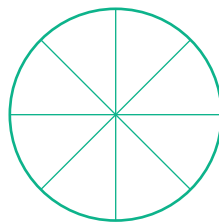
1 cut



2 cuts



3 cuts



4 cuts

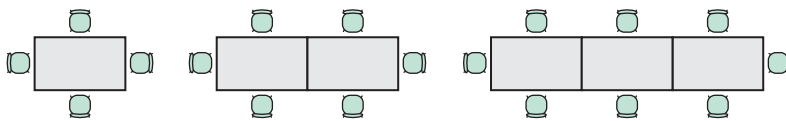
**A**  $p = 2c$

**B**  $c = 2p$

**C**  $p = c + 2$

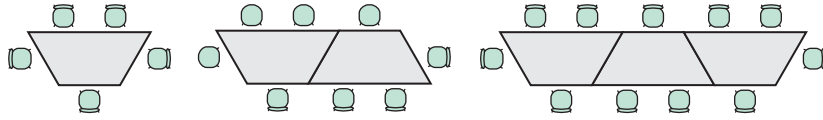
**D**  $c = p + 2$

- 8** In a restaurant, chairs are placed around rectangular tables as shown below. **PS R C**



- a** Find the number of chairs required for:
- i** 2 tables      **ii** 3 tables      **iii** 4 tables      **iv** 12 tables
- b** Find a formula that relates the number of chairs,  $C$ , to the number of tables,  $T$ .
- c** How many chairs would be needed for a row of 50 tables?
- d** A booking is made at the restaurant for 30 people. How many tables are needed?

- 9 Students arrange themselves around desks ready for group work in the classroom. The seating arrangement for  $n$  students at  $d$  desks is shown. **PS R C**



- a Find the number of students who can be seated around a row of:
- i 2 desks      ii 3 desks      iii 4 desks      iv 10 desks
- b Find a formula that relates the number of students,  $n$ , to the number of desks,  $d$ .
- c Find the number of students that could be seated around 18 desks.
- d Class 8Y has 29 students. How many desks are needed for them?

## 12.04 The number plane



The number plane



The number plane



The number plane



Plane grid 2



Number plane review



Number plane grid paper



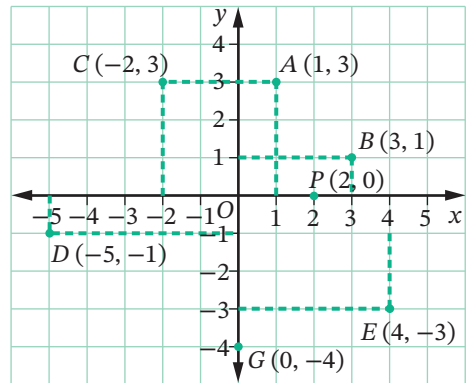
A page of number planes



Plotting points and lines

A **number plane** is a grid made from a horizontal number line called the  **$x$ -axis** and a vertical number line called the  **$y$ -axis**. The number plane is also called a **Cartesian plane**, named after René Descartes (pronounced 'Ren-ay Day-cart'), a French mathematician and philosopher who developed the idea.

Points on the number plane are located using a pair of **coordinates** or an **ordered pair**. Note that order is important with coordinates:  $A(1, 3)$  is not the same as  $B(3, 1)$ .



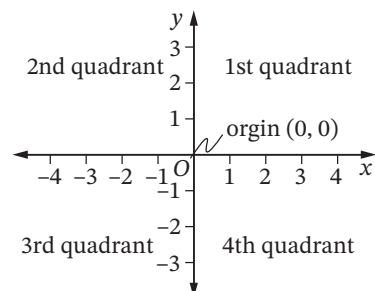
### Points on the number plane

In any ordered pair, the first number is called the  **$x$ -coordinate** and the second number is called the  **$y$ -coordinate**.

The point  $(0, 0)$  is called the **origin**.

To locate a point, always start from  $(0, 0)$ : the  $x$ -coordinate tells you how far to move *across*, the  $y$ -coordinate tells you how far to move *up* or *down*.

The number plane is divided evenly into 4 regions called **quadrants**.



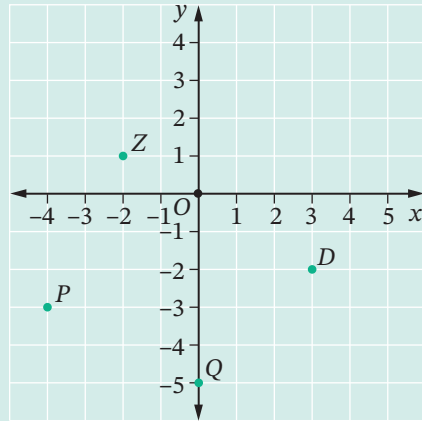
Foundation Standard Complex

## Example 5

- a Write the coordinates of point  $D$  and state what quadrant it is in.
- b Write the point with coordinates  $(-4, -3)$  and state what quadrant it is in.
- c Write the coordinates of point  $Q$ .

### Solution

- a  $D(3, -2)$  is in the 4th quadrant.
- b  $(-4, -3)$  is the point  $P$ , which is in the 3rd quadrant
- c Point  $Q$  is 0 on the  $x$ -axis and  $-5$  on the  $y$ -axis, so its coordinates are  $(0, -5)$ .



Coordinates code puzzle



Graph coordinates



Plotting points



Points plotter



Number plane writing activity

12.04

## EXERCISE 12.04 ANSWERS ON P. 582

### The number plane U F R C

Questions 1 to 5 refer to this diagram.

- 1 Name the point with each pair of coordinates.

- a  $(2, 1)$                       b  $(-3, 1)$
- c  $(5, 2\frac{1}{2})$                       d  $(5, -2)$
- e  $(0, 3)$                           f  $(0, -4)$
- g  $(2, 0)$                           h  $(-1\frac{1}{2}, 0)$
- i  $(-4, 2)$                         j  $(4, -1)$

- 2 Write the coordinates of each point. **C**

- a  $A$                               b  $B$                               c  $C$
- d  $D$                               e  $E$                               f  $F$
- g  $G$                               h  $H$                               i  $I$                               j  $J$

- 3 In which quadrant does each point lie? **C**

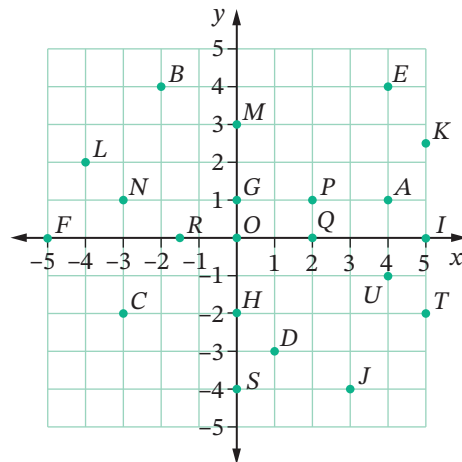
- a  $N$                               b  $T$                               c  $E$                               d  $L$                               e  $C$

- 4 List all of the points that are on the  $x$ -axis and write their coordinates. **C**

- 5 List all of the points that are on the  $y$ -axis and write their coordinates. **C**

- 6 If a point has a positive  $x$ -coordinate and a negative  $y$ -coordinate, which quadrant is it in? Select the correct answer **A, B, C** or **D**. **R**

- A** 1st quadrant                      **B** 2nd quadrant                      **C** 3rd quadrant                      **D** 4th quadrant



EXAMPLE 5



**7** Determine the quadrant in which each point lies. **R C**

- a**  $(3, -5)$       **b**  $(-2, -4)$       **c**  $(-8, 1)$       **d**  $(5, 4)$

**8 a** Draw a number plane, with both axes extending from  $-8$  to  $8$ , or use the link to print Plane grid 2.

**b** Plot and label these points.

- $A(3, 0)$        $B(-2, 2)$        $C(4, 5)$        $D(3\frac{1}{2}, -5)$        $E(-1, 7)$   
 $F(0, -4)$        $G(-6, 0)$        $H(\frac{1}{2}, -7\frac{1}{2})$        $I(-2, 6)$        $J(7, 0)$

**9 a** Draw a number plane, with both axes extending from  $-3$  to  $3$ , or use the link to print Number plane grid paper.

**b** Mark these points.

- $A(0, 1)$ ,       $B(2, -1)$ ,       $C(-1, 2)$ ,       $D(-2, 3)$ ,       $E(3, -2)$        $F(1, 0)$ .

**c** What do you notice about the points? Check this with your ruler. **c**

**d** Points that lie on the same straight line are said to be **collinear**. Write down the coordinates of any 4 points on a number plane that are collinear.



## Did you know?



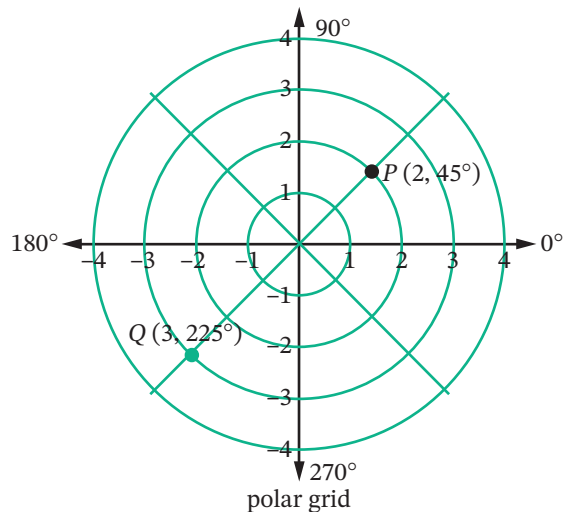
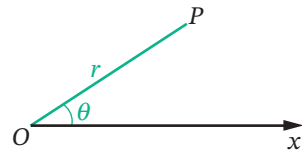
### Polar coordinates

The number plane is not the only way to represent the position of a point. The **polar coordinates** of a point give its position in terms of a distance from the origin and angle measured from the  $x$ -axis.

We say that  $(r, \theta)$  are the polar coordinates of the point  $P$ , where  $r$  is the distance  $P$  is from the origin  $O$  and  $\theta$  is the angle between  $Ox$  and  $OP$ . A polar grid shows both distance and angle. For example, the grid below shows the coordinates of  $P$  as  $(2, 45^\circ)$  and  $Q$  as  $(3, 225^\circ)$ .

Some of the real-life uses of polar coordinates include avoiding collisions between vessels and other ships, calculating the flow of ground water and guiding industrial robots.

**On polar grid paper, plot the points  $L(2, 90^\circ)$ ,  $M(4, 135^\circ)$  and  $N(0, 270^\circ)$**



### The unitary method with percentages

The unitary method is used when you are only given a *percentage* of an amount and you need to find the amount. It is called the unitary method because we find 1% of the amount first, then multiply that by 100 to find the whole (100%).

1 Study each example.

a If 8% of a number is 24, what is the number?

$$8\% \text{ of the number} = 24$$

$$\therefore 1\% \text{ of the number} = 24 \div 8 = 3$$

$$\therefore 100\% \text{ of the number} = 3 \times 100 = 300.$$

The number is 300. *Check:*  $8\% \times 300 = 24$

b If 15% of an amount is \$90, what is the whole amount?

$$15\% \text{ of the amount} = \$90$$

$$\therefore 1\% \text{ of the amount} = \$90 \div 15 = \$6$$

$$\therefore 100\% \text{ of the amount} = \$6 \times 100 = \$600.$$

The amount is \$600. *Check:*  $15\% \times \$600 = \$90$

2 Find the whole amount if:

a 5% of the amount is \$35

b 11% of the amount is \$88

c 20% of the amount is 80

d 6% of the amount is 42

e 90% of the amount is \$270

f 15% of the amount is \$60

g 40% of the amount is 100

h 120% of the amount is \$360

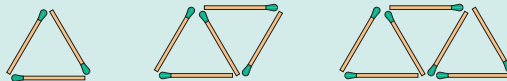
i 25% of the amount is \$75

j 8% of the amount is 40

## Graphing number patterns

### Example 6

Consider this pattern of matchsticks used to make triangles.



a Copy and complete the table of values for this pattern.

No. of triangles, $x$	1	2	3	4	5
No. of matchsticks, $y$					

b Find the formula for this table of values.

c Graph this table of values.



Number plane grid paper



A page of number planes



Graphing tables of values

## Solution

**a**

No. of triangles, $x$	1	2	3	4	5
No. of matchsticks, $y$	3	5	7	9	11

**b** Bottom row of  $y$ -values increase by 2, so the multiplier in the formula is 2, and the equation is of the form  $y = 2x + \underline{\hspace{1cm}}$ .

Choose (2, 5) from the table to find the missing number.

When  $x = 2$ ,  $y = 2 \times 2 + 1 = 5$ .

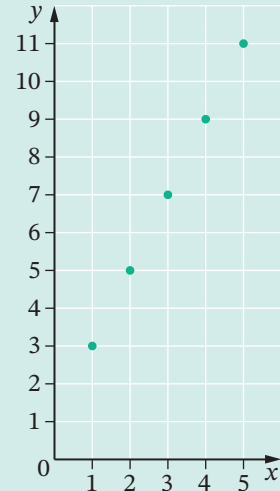
The formula is  $y = 2x + 1$ .

**c** Each column in the table forms an ordered pair.

No. of triangle, $x$	1	2	3	4	5
No. of matchsticks, $y$	3	5	7	9	11

$(1, 3)$        $(2, 5)$        $(3, 7)$        $(4, 9)$        $(5, 11)$

Graph each ordered pair on a number plane.



Note that the points are **collinear**, that is, they lie on a straight line. When this happens, we say that the relationship or pattern is **linear**.

- **linear** = involving a line
- **collinear points** = points that lie on a straight line

### EXERCISE 12.05 ANSWERS ON P. 582

## Graphing number patterns **U F R C**

**1** Find the formula for this table of values. Select the correct answer **A**, **B**, **C** or **D**. **R C**

$x$	1	2	3	4	5
$y$	2	6	10	14	18

- A**  $y = 3x$       **B**  $y = 2x + 2$       **C**  $y = 4x - 2$       **D**  $y = 4x$

**2** This table of values shows the height of a tree as it ages. Graph the values and determine whether there is a linear pattern between the age and height of the tree. **R C**

Age, $x$ (years)	5	10	15	20	25
Height, $y$ (metres)	10	12.5	17.5	21	26

EXAMPLE  
6

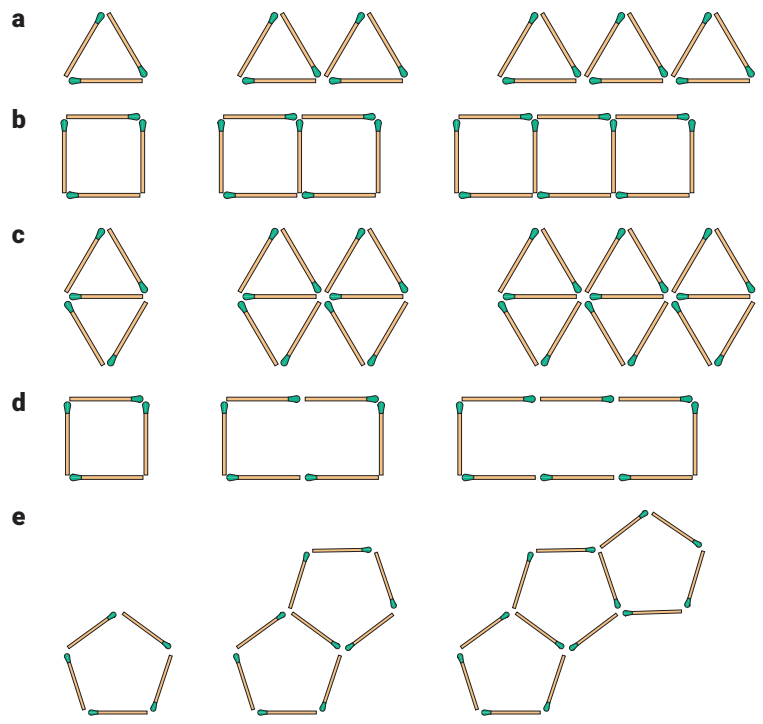
**3** For each pattern below: **R C**

**i** copy and complete this table

No. of shapes, $x$	1	2	3	4	5
No. of matchsticks, $y$					

**ii** find the formula for the table of values

**iii** graph the table of values.



**4** Does each graph in question 3 show a linear relationship?

**5** Cans in supermarkets are often stacked neatly in triangles. Consider the relationship between the number of cans on the bottom row,  $x$  and the total number of cans,  $y$ . **R C**



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**a** Copy and complete this table.

Length of triangle, $x$	1	2	3	4	5	6	7
No. of cans, $y$							

**b** Graph this table on a number plane.

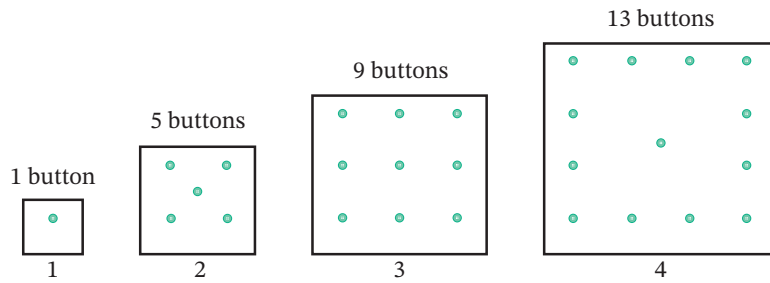
**c** Is the relationship linear?

**d** Which formula matches the table of values?

Select the correct answer **A**, **B** or **C**.

**A**  $y = x(x - 1)$     **B**  $y = \frac{x+1}{2}$     **C**  $y = \frac{x(x+1)}{2}$

6 The following pattern is made with buttons. **R C**



- Complete a table of values showing the relationship between the length of the square ( $S$ ) and the number of buttons ( $B$ ).
- Find a rule for  $S$  in terms of  $B$ .
- Use your rule to find the number of buttons required to make a pattern for a square of length:
  - 10
  - 24
- Kovo has 106 buttons. What is the length of the biggest complete square that she can build?

## 12.06 Graphing linear equations



Number plane grid paper

A **linear equation** is an equation or formula whose graph on a number plane is a **straight line**.



Matching linear equations

### Graphing linear equations

- complete a table of values
- graph the table of values on a number plane
- rule a line through the points and label the line with the equation.



Graphing functions

### Example 7

Graph  $y = x + 1$  on a number plane.



Graphing linear equations

### Solution

Complete a table of values. Choose  $x$ -values close to 0 for easy calculation and graphing.

$x$	-2	-1	0	1	2
$y$	-1	0	1	2	3

Graph the table of values on a number plane.

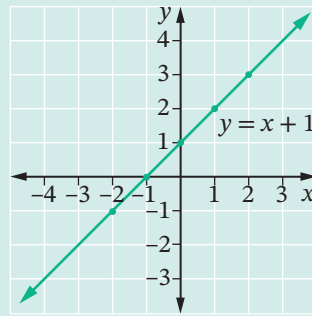
Rule a straight line through the points, place arrows at each end, and label the line with its equation.



A page of lines

Note:

- Every pair of values that follows the linear equation  $y = x + 1$  lies on this line, for example,  $(3, 4)$ ,  $(-4, -3)$ ,  $(1\frac{1}{2}, 2\frac{1}{2})$ ,  $(5, 6)$ ,  $(100, 101)$ ,  $(-\frac{1}{2}, \frac{1}{2})$ .
- Every point on the line follows the linear equation  $y = x + 1$ .
- There are an infinite number of points that satisfy the rule.



For these reasons, the line is infinite so we must draw arrows at each end.



Linear functions



Graphing straight lines:  
Finding intercepts



Linear function plotter

12.06

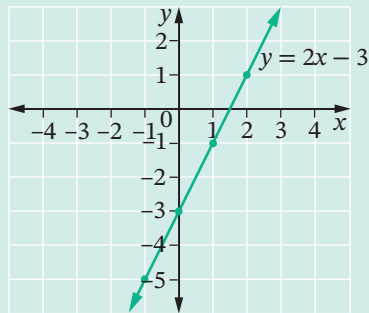
## Example 8

- a** Graph  $y = 2x - 3$  on a number plane.
- b** Test whether the point  $(5, 8)$  lies on the line  $y = 2x - 3$ .

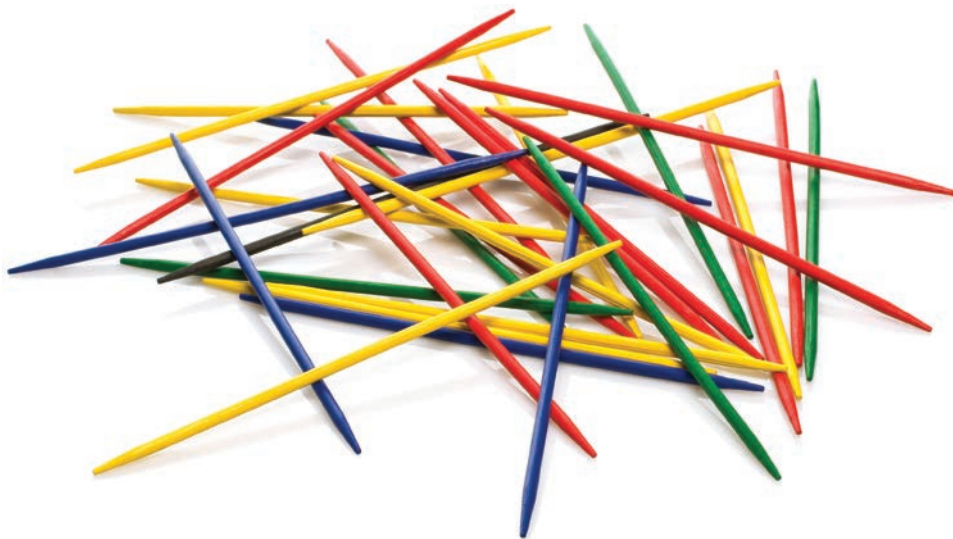
### Solution

**a**

x	-1	0	1	2
y	-5	-3	-1	1



- b** We could extend the line  $y = 2x - 3$  to see if  $(5, 8)$  lies on it, but a simpler way is to check whether  $(5, 8)$  follows or **satisfies** the formula  $y = 2x - 3$ . If it does, then when  $x = 5$ ,  $y = 8$ .
- When  $x = 5$ ,  $y = 2 \times 5 - 3 = 7 \neq 8$
- $\therefore (5, 8)$  does not lie on the line. (In fact,  $(5, 7)$  does)



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## Graphing linear equations U F R C

EXAMPLE  
7

WS  
Number  
plane grid  
paper

WS  
A page of  
number  
planes

EXAMPLE  
8

**1** For each linear equation, copy and complete the table of values and graph the equation on a number plane. **C**

**a**  $y = x + 3$

x	-1	0	1	2
y				

**b**  $y = x - 2$

x	0	1	2	3
y				

**c**  $y = 2x$

x	-1	0	1	2
y				

**d**  $y = \frac{x}{2}$

x	-1	0	1	2
y				

**e**  $y = 6 - x$

x	0	1	2	3
y				

**f**  $y = x$

x	-1	0	1	2
y				

**2** Graph each linear equation on a number plane.

**a**  $y = 2x + 1$

**b**  $y = 2x - 1$

**c**  $y = 3x$

**d**  $y = 3x - 2$

**e**  $y = 2x + 4$

**f**  $y = 3x - 5$

**3** Look at the graphs you drew in question 2. Why are they called **increasing graphs**? **C**

**4** Graph each linear equation on a number plane.

**a**  $y = -x$

**b**  $y = -x + 5$

**c**  $y = 4 - x$

**d**  $y = -x - 3$

**e**  $y = 1 - 3x$

**f**  $y = -2x$

**5** Look at the graphs you drew in question 4. Why are they called **decreasing graphs**? **C**

**6** By looking at each linear equation, predict whether its graph will be increasing or decreasing. **R C**

**a**  $y = 5x - 4$

**b**  $y = x + 9$

**c**  $y = 3 - x$

**d**  $y = 2x$

**e**  $y = -x + 5$

**f**  $y = -3x - 1$

**7** Test whether each point lies on the graph of the given linear equation by: **R**

**i** substituting its coordinates into the linear equation

**ii** examining the graphs of the lines you drew in questions 1 and 2

**a**  $(4, 7) y = x + 3$

**b**  $(-1, -1) y = x - 2$

**c**  $(5, 1) y = 6 - x$

**d**  $(-3, -5) y = 2x + 1$

**e**  $(4, 10) y = 3x - 2$

**f**  $(-2, 1) y = 2x + 4$

**8** On which line does the point  $(1, 3)$  lie? Select the correct answer **A, B, C** or **D**.

**A**  $y = -x + 3$

**B**  $y = x + 4$

**C**  $y = 2 - 2x$

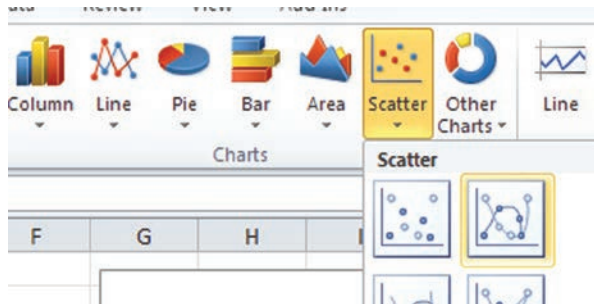
**D**  $y = 2x + 1$

## Technology

### Graphing linear equations

We are going to use a spreadsheet to graph  $y = 2x - 3$ .

- 1 Enter these values into a spreadsheet.
- 2 In cell B2, enter the formula  $=2*A2-3$  to evaluate  $2x - 3$  and use **Fill Down** to copy the formula down to cell B11.
- 3 Highlight columns A and B, then click **Insert** and **Scatter** (with **Smooth Lines and Markers**).



	A	B
1	x	y
2	-4	
3	-3	
4	-2	
5	-1	
6	0	
7	1	
8	2	
9	3	
10	4	
11	5	
12		

- 4 Click the bottom-right corner of the graph border to enlarge the graph.
- 5 Use your graph to predict the value of  $y$  when:
  - a  $x = 5$
  - b  $x = -6$
  - c  $x = -1.5$
- 6 Use your graph to predict the value of  $x$  when:
  - a  $y = 11$
  - b  $y = -9$
  - c  $y = 14$

## Investigation

### Special lines

- 1 Draw a number plane with both axes extending from  $-4$  to  $4$  and graph each table of values on it.

**a**

x	-1	0	2	3
y	2	2	2	2

**b**

x	-2	0	1	3
y	-1	-1	-1	-1

**c**

x	2	-1	0	-4
y	1	1	1	1

**d**

x	-2	3	0	4
y	-3	-3	-3	-3

- 2
  - a What do all of the lines in question 1 have in common?
  - b What do all of the tables of values in question 1 have in common?
  - c The formula for the table of values and line in question 1 a is  $y = 2$ . It does not have an  $x$  in it because the value of  $y$  does not depend on  $x$ , but is always equal to 2. What are the formulas for the other 3 lines in question 1?



**3** Draw another number plane and graph each table of values on it.

**a**

x	1	1	1	1
y	-3	-2	0	2

**b**

x	-2	-2	-2	-2
y	-2	0	1	3

**c**

x	3	3	3	3
y	-3	2	-2	0

**d**

x	-3	-3	-3	-3
y	-4	-1	2	4

- 4 a** What do all of the lines in question **3** have in common?  
**b** What do all of the tables of values in question **3** have in common?  
**c** The formula for the table of values and line in question **3 a** is  $x = 1$ .  
 What are the formulas for the other 3 lines in question **3**?

## 12.07 Finding the equation of a line



Finding linear equations



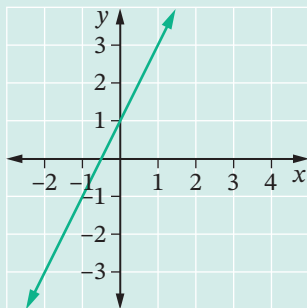
Matching linear equations



Finding the equation of a line

### Example 9

Find the equation of this line.



### Solution

First draw a table of values using the coordinates of the points on the line, with consecutive  $x$ -values.

x	-2	-1	0	1
y	-3	-1	1	3

$\underbrace{\hspace{1.5em}}_{+2}$ 
 $\underbrace{\hspace{1.5em}}_{+2}$ 
 $\underbrace{\hspace{1.5em}}_{+2}$

The  $y$ -values go up by 2, so the multiplier is 2, and the equation is of the form  $y = 2x$  \_\_\_\_.

Choose the point (1, 3) to find the missing number.

When  $x = 1$ ,  $y = 2 \times 1 + 1 = 3$ .

So the equation is  $y = 2x + 1$ .

(Check that this is also true for (0, 1)).

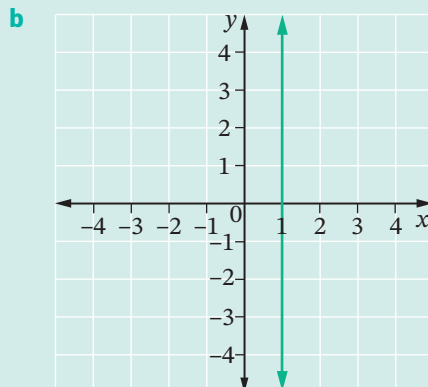
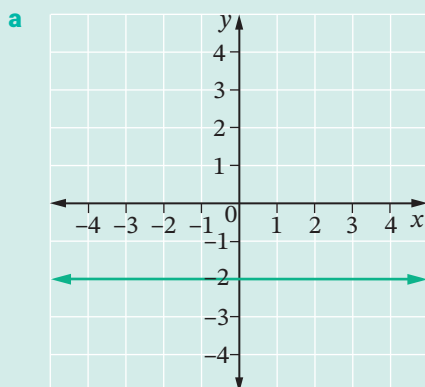
## Horizontal and vertical lines

A **horizontal** line is flat and runs across (sideways). The word ‘horizontal’ comes from ‘horizon,’ the flat line that is the edge of the Earth as far as your eye can see.

A **vertical** line runs up and down, at right angles to a horizontal line. The word ‘vertical’ means upright. For example, vertical blinds are made up of columns of slats.

### Example 10

Find the equation of each line.



### Solution

**a** This is a **horizontal** line.

<b>x</b>	-1	0	1	2
<b>y</b>	-2	-2	-2	-2

$y$  is always equal to  $-2$ .

So the equation is  $y = -2$ .

**b** This is a **vertical** line.

<b>x</b>	1	1	1	1
<b>y</b>	0	1	2	3

$x$  is always equal to  $1$ .

So the equation is  $x = 1$ .

## Horizontal and vertical lines

The **equation of a horizontal line** is of the form  $y = c$  (where  $c$  is a constant number).

The **equation of a vertical line** is of the form  $x = c$  (where  $c$  is a constant number).

### EXERCISE 12.07 ANSWERS ON P. 585

### Finding the equation of a line **UFRC**

- 1** Find the equation that describes this table of values. Select the correct answer **A, B, C** or **D**.

<b>x</b>	-1	0	1	2
<b>y</b>	3	1	-1	-3

**A**  $y = 2x + 1$

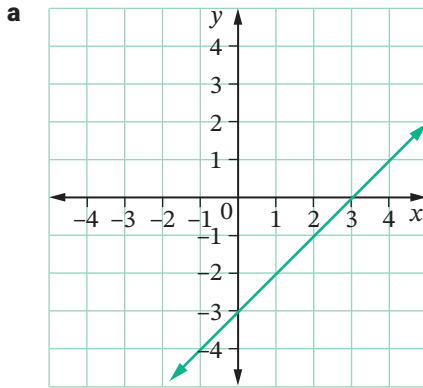
**B**  $y = 2x - 1$

**C**  $y = x - 2$

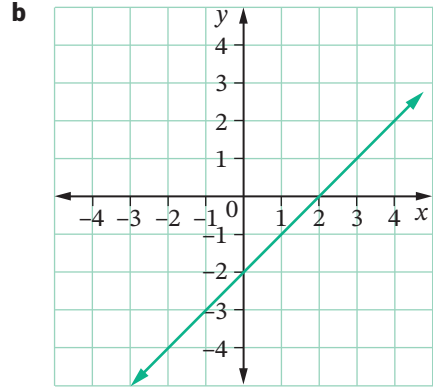
**D**  $y = -2x + 1$

EXAMPLE  
9

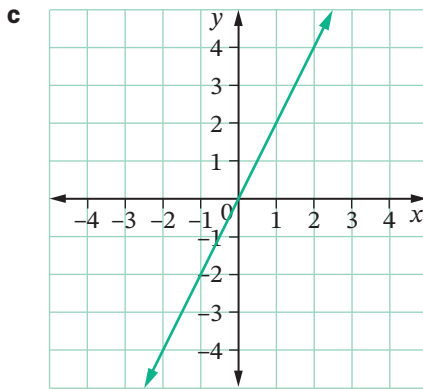
**2** For each graph, copy and complete the table of values and use it to find the equation of the line.



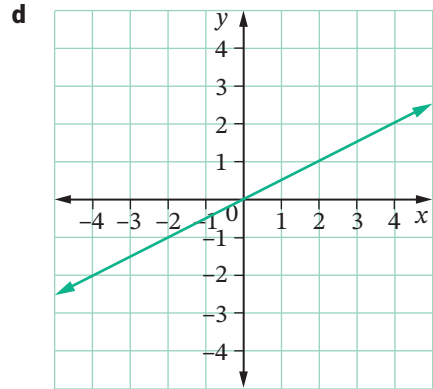
x	-1	0	1	2
y				



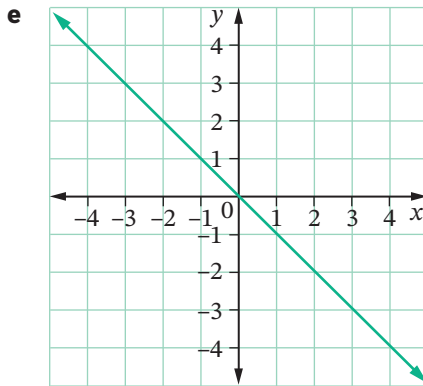
x	-2	-1	0	1
y				



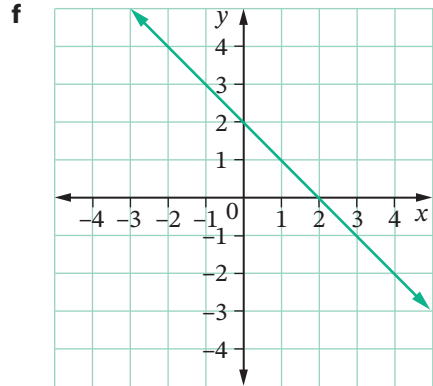
x	-1	0	1	2
y				



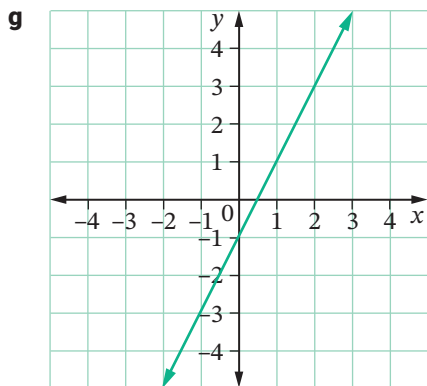
x	0	1	2	3
y				



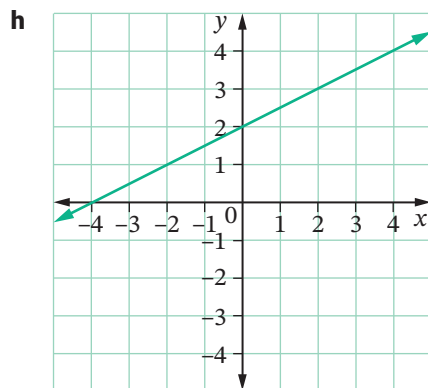
x	-1	0	1	2
y				



x	-3	-2	-1	0
y				



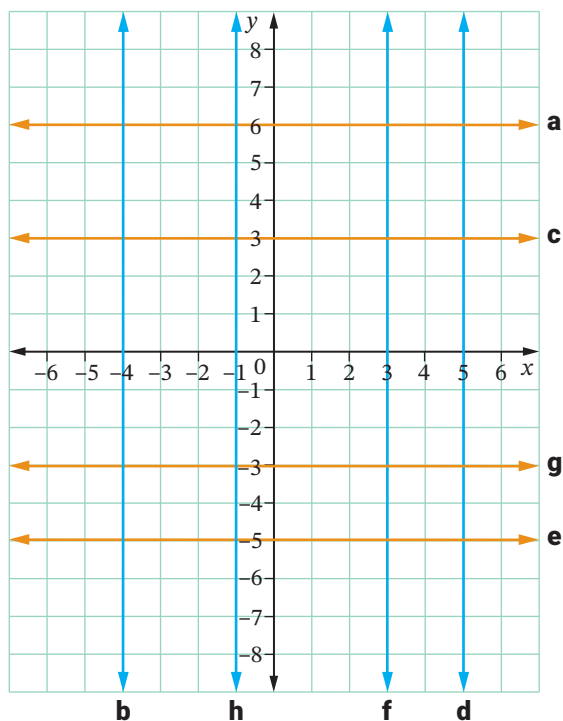
x	-1	0	1	2
y				



x	-2	-1	0	1
y				

- 3 a** What do the lines represented by  $x = 2$ ,  $x = 5$ ,  $x = -6$  and  $x = \frac{1}{2}$  have in common? **R C**
- b** What do the lines represented by  $y = 2$ ,  $y = 5$ ,  $y = -6$  and  $y = \frac{1}{2}$  have in common?
- 4** Write down the equation of each line in the diagram below.

EXAMPLE 10



- 5** Graph each line on the same number plane.

**a**  $x = 4$

**b**  $x = -2$

**c**  $y = 5$

**d**  $y = -1$

**e**  $x = 1$

**f**  $y = -3$

**g**  $y = 4$

**h**  $x = -3$



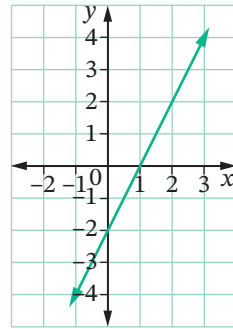
**6** What is the equation of this line? Select **A**, **B**, **C** or **D**.

**A**  $y = -x + 2$

**B**  $y = 2x - 2$

**C**  $y = 2 - 2x$

**D**  $y = x - 2$



**7** Harrison read the following points from a line that he drew.

$$(-4, 0), (-3, \frac{1}{2}), (-2, 1), (-1, 1\frac{1}{2})$$

Find the equation of the line that Harrison drew. **R C**

## Technology

### Comparing linear equations

In this activity, we will use a spreadsheet to compare the graphs of  $y = x$ ,  $y = 2x$  and  $y = 3x$ .

**1** Set up this spreadsheet.

	A	B	C	D
1	x	y = x	y = 2x	y = 3x
2	-2			
3	-1			
4	0			
5	1			
6	2			
7				

**2** In cell B2 enter **=A2** to calculate  $y = x$ , then use **Fill Down** to copy this formula down to B6.

**3** In cell C2 enter **=2\*A2** to calculate  $y = 2x$ , then **Fill Down** to C6.

**4** In cell D2, write a formula to calculate  $y = 3x$ , then **Fill Down** to D6.

**5** Highlight all cells. Click **Insert** and **Scatter (Smooth Lines and Markers)** to graph all 3 lines on the same axes.

**6** Compare the 3 lines. What is the same and what is different? Describe any patterns and important features. Write your answers on your spreadsheet or in your workbook.

**7** Set up a similar spreadsheet for each set of 3 linear equations shown below. Use appropriate formulas to complete each column and graph each set of 3 lines. Compare the 3 lines in each set and describe the pattern and important features of each set.

**a**

G	H	I	J
x	y = -x	y = -2x	y = -5x
-2			
-1			
0			
1			
2			

**b**

L	M	N	O
x	y = x + 1	y = x + 2	y = x - 4
-2			
-1			
0			
1			
2			

**c**

Q	R	S	T
x	y = -x + 1	y = -x + 2	y = -x - 4
-2			
-1			
0			
1			
2			

# Comparing linear equations

12.08

A **linear equation** such as  $y = 2x + 1$  is made up of 2 terms:

- the **variable term**, or the term with the  $x$
- the **constant term**, the number without the  $x$ .

The number in front of the  $x$  is called the **coefficient** of  $x$ . For example,

$$y = 2x + 1$$

The coefficient of  $x$  is 2.      The constant term is 1.



Matching  
linear  
equations



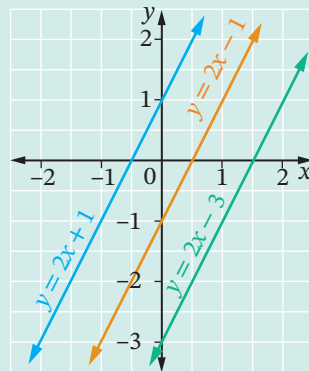
Linear  
functions

12.08

## Example 11

The lines  $y = 2x + 1$ ,  $y = 2x - 1$   
and  $y = 2x - 3$  are graphed.

- How are the lines similar?
- How are the linear equations similar?
- The  $y$ -intercept of a line is the value at which the line crosses the  $y$ -axis. Find the  $y$ -intercept of each line in the diagram.
- How are the linear equations different?



## Solution

- The lines point in the same direction and have the same slope, they are parallel.
- The linear equations all have the same coefficient of  $x$ , namely, 2.
- $y = 2x + 1$ :  $y$ -intercept = 1  
 $y = 2x - 1$ :  $y$ -intercept =  $-1$   
 $y = 2x - 3$ :  $y$ -intercept =  $-3$
- The linear equations have different constant terms: 1,  $-1$ ,  $-3$ .

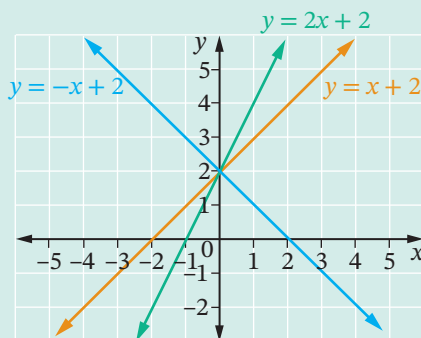


iStock.com/Rahimiro Lotufo Neto

The Octavio Frias de Oliveira bridge in São Paulo, Brazil.

## Example 12

The lines  $y = x + 2$ ,  $y = -x + 2$  and  $y = 2x + 2$  are graphed.



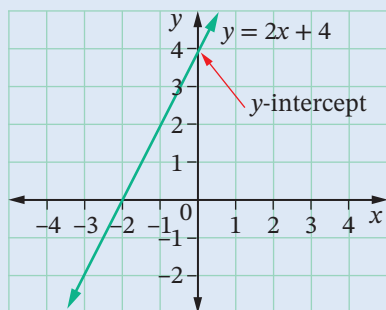
- Find the  $y$ -intercept of each line.
- How are the linear equations similar?
- How are the lines different?
- How are the linear equations different?
- How does the coefficient of  $x$  in a linear equation affect its graph?
- How does the constant term in a linear equation affect its graph?

### Solution

- $y = -x + 2$ :  $y$ -intercept = 2  
 $y = 2x + 2$ :  $y$ -intercept = 2  
 $y = x + 2$ :  $y$ -intercept = 2
- The linear equations all have the same constant term, 2.
- The lines point in different directions: they are not parallel.
- The linear equations have different coefficients of  $x$ :  $-1$ ,  $2$  and  $1$ .
- The coefficient of  $x$  affects the direction of the graph.
- The constant term gives the  $y$ -intercept of the graph.

## Features of a linear equation

- In a linear equation, the **coefficient of  $x$**  gives the **direction** of the line.
- If 2 or more lines have the same coefficient of  $x$  in their equations, then they are parallel.
- The **constant term** gives the  **$y$ -intercept** of the line.



### EXERCISE 12.08 ANSWERS ON P. 586

## Comparing linear equations U F R C

The exercise is best completed using dynamic geometry or graphing software.

- For each linear equation, state the coefficient of  $x$  and the constant term. **C**

**a**  $y = 4x + 1$

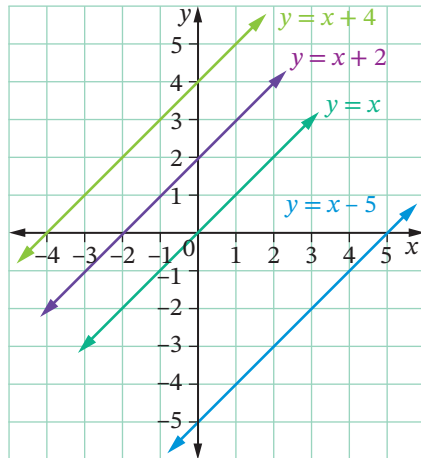
**b**  $y = \frac{1}{2}x - 3$

**c**  $y = x - 5$

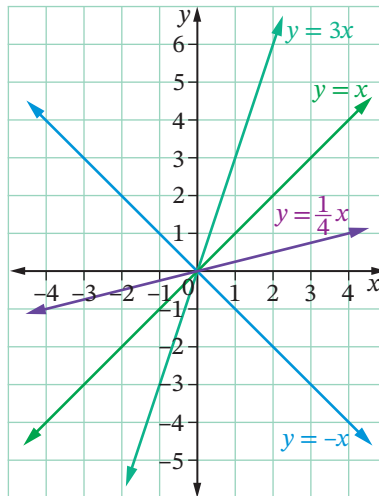
**d**  $y = -x + 6$

EXAMPLE  
11

- 2** The lines  $y = x + 4$ ,  $y = x + 2$ ,  $y = x$  and  $y = x - 5$  are graphed. **R C**
- How are the lines similar?
  - How are the linear equations similar?
  - How are the lines different?
  - How are the linear equations different?



- 3** The lines  $y = \frac{1}{4}x$ ,  $y = x$ ,  $y = 3x$  and  $y = -x$  are graphed. **R C**
- How are the lines similar?
  - How are the linear equations similar?
  - How are the lines different?
  - How are the linear equations different?
  - Which line is the steepest? What is the coefficient of  $x$  in its equation?
  - Which line is the least steep? What is the coefficient of  $x$  in its equation?
  - How does the size of the coefficient of  $x$  in a linear equation affect the steepness of its line?



- 4** Graph  $y = 3x$ ,  $y = 3x - 1$  and  $y = 3x + 2$  on the same number plane. **R C**
- What is the same about all 3 lines?
  - What is the same about all 3 linear equations?
  - What is different about the lines?
  - What is different about the linear equations?
- 5** Graph  $y = x + 1$ ,  $y = 3x + 1$  and  $y = \frac{1}{2}x + 1$  on the same number plane. **R C**
- What is the same about all 3 lines?
  - What is the same about all 3 linear equations?
  - What is different about the lines?
  - What is different about the linear equations?
  - What is the coefficient of  $x$  in the equation with the steepest line?

EXAMPLE  
12

12.08



**6** Graph  $y = -x - 2$ ,  $y = -2x - 2$  and  $y = -\frac{1}{2}x - 2$  on the same number plane. **R C**

- a How are the lines similar?
- b How are the linear equations similar?
- c How are the lines different?
- d How are the lines different to the lines in question 5?
- e What is the coefficient of  $x$  in the equation with the least steepest line?

**7** The graphs in question 5 are all increasing, while the graphs in question 6 are all decreasing. What feature of their equations indicates this? **R C**

**8** Which equation below when graphed gives a decreasing line with a  $y$ -intercept of 5? Select the correct answer **A, B, C** or **D**. **R C**

- A**  $y = 5x$       **B**  $y = 5 - x$       **C**  $y = 5x - 5$       **D**  $y = -5x$

**9** Copy and complete each statement. **R C**

- a The equations of parallel lines have the same \_\_\_\_\_.
- b Linear equations with the same constant term have graphs that have the same \_\_\_\_\_.
- c Linear equations with a negative coefficient of  $x$  have graphs that are \_\_\_\_\_.
- d The equations of lines with the same  $y$ -intercept have the same \_\_\_\_\_.
- e The equations of lines that are decreasing have a \_\_\_\_\_ coefficient of  $x$ .

**10** Graph each **non-linear equation** after completing the table of values given. **R C**

a  $y = x^2$

x	-3	-2	-1	0	1	2	3
y							

b  $y = \sqrt{x}$

x	0	1	2	4	5	7	9
y							

c  $y = x^3$

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y									

d  $y = \frac{1}{x}$

x	0.5	1	1.5	2	2.5	3	3.5	4	4.5
y									

**11 a** Why do you think the equations graphed in question 10 are called non-linear equations? **R C**

**b** How are the non-linear equations algebraically different to the linear equations?

# Solving linear equations graphically

12.09

Equations can be solved **algebraically** using the balancing and backtracking methods. However, we can also solve an equation **graphically** by first graphing it on the number plane.

## Example 13

Solve the equation  $2x - 3 = 5$  graphically.

### Solution

Graph  $y = 2x - 3$  on a number plane (the LHS of the equation).

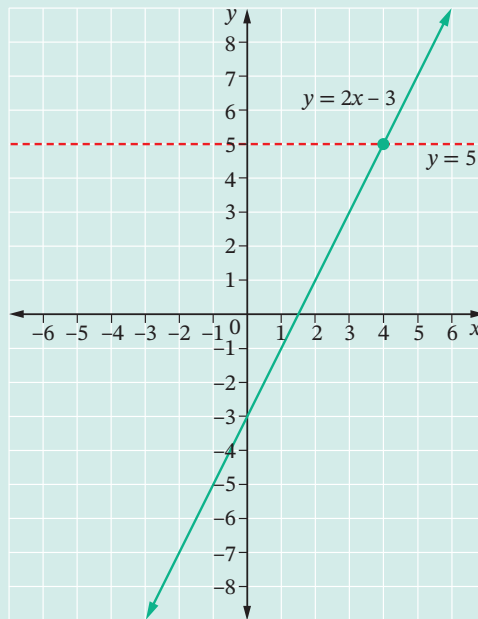
x	-1	0	1	2
y	-5	-3	-1	1

For each point in the table and on the line, the y-coordinate is the value of  $2x - 3$ . So to solve  $2x - 3 = 5$ , we need to find the point whose y-coordinate is 5.

Draw a dotted horizontal line at  $y = 5$  (shown in red on the diagram) and read off the coordinates for the point on  $y = 2x - 3$  crossed by this line.

This point is  $(4, 5)$ , which means that the solution to  $2x - 3 = 5$  is  $x = 4$ .

Check:  $2 \times 4 - 3 = 5$



## EXERCISE 12.09 ANSWERS ON P. 587

### Solving linear equations graphically UF

This exercise is best completed using dynamic geometry software or a graphing website.

**1** Use the graph in Example 13 above to solve each equation below. Check your solutions.

**a**  $2x - 3 = 3$

**b**  $2x - 3 = -7$

**c**  $2x - 3 = 0$

**2** Graph  $y = 2x + 1$  and use it to solve each equation below graphically.

**a**  $2x + 1 = 7$

**b**  $2x + 1 = 10$

**c**  $2x + 1 = -5$

**3** Solve each equation in question 2 algebraically.

**4** Graph  $y = 2x - 1$  and use it to solve each equation below graphically.

**a**  $2x - 1 = -5$

**b**  $2x - 1 = -1$

**c**  $2x - 1 = 4$



Number plane grid paper



A page of number planes



EXAMPLE 13



- 5** Graph  $y = 3x - 2$  and use it to solve each equation graphically.
- a**  $3x - 2 = 7$                       **b**  $3x - 2 = -11$                       **c**  $3x - 2 = -2$
- 6** Graph  $y = -x + 1$  and use it to solve each equation graphically.
- a**  $-x + 1 = -3$                       **b**  $-x + 1 = 4$                       **c**  $-x + 1 = -1$
- 7** Graph  $y = -2x - 1$  and use it to solve each equation graphically.
- a**  $-2x - 1 = -9$                       **b**  $-2x - 1 = 0$                       **c**  $-2x - 1 = 3$
- 8** Graph  $y = \frac{1}{2}x + 3$  and use it to solve each equation graphically.
- a**  $\frac{1}{2}x + 3 = 5$                       **b**  $\frac{1}{2}x + 3 = 1$                       **c**  $\frac{1}{2}x + 3 = 2$

## Technology

### Intersecting lines

We will graph the 2 straight lines  $y = x + 5$  and  $y = 3 - x$  using a spreadsheet to find their point of intersection.

- Set up this spreadsheet.
- In cell B2, enter  $=A2+5$ , then **Fill Down** to cell B10.
- In cell C2, enter  $=3-A2$ , then **Fill Down** to cell C10.
- Highlight all 3 columns and click **Insert** and **Scatter (Smooth Lines and Markers)**.
- From the graph, read the point of intersection of the 2 straight lines.
- Find the point of intersection for each pair of straight lines below, by setting up a separate spreadsheet for each pair of lines and using **Insert** and **Scatter (Smooth Lines and Markers)** (as above) to graph each pair of lines on the same set of axes.

	A	B	C
1	x	$y = x + 5$	$y = 3 - x$
2	-4		
3	-3		
4	-2		
5	-1		
6	0		
7	1		
8	2		
9	3		
10	4		

**a**

	A	B	C
1	x	$y = 2x - 2$	$y = -x + 7$
2	-4		
3	-3		
4	-2		
5	-1		
6	0		
7	1		
8	2		
9	3		
10	4		

**b**

	A	B	C
1	x	$y = 3x - 6$	$y = -4x - 6$
2	-4		
3	-3		
4	-2		
5	-1		
6	0		
7	1		
8	2		
9	3		
10	4		

**c**

	A	B	C
1	x	$y = 10 - 2x$	$y = 2x$
2	-4		
3	-3		
4	-2		
5	-1		
6	0		
7	1		
8	2		
9	3		
10	4		

## Example 14

Graph  $y = 2x - 2$  and  $y = x + 1$  on the same number plane and use the graphs to solve the equation  $2x - 2 = x + 1$ .

### Solution

$$y = 2x - 2$$

x	-1	0	1	2
y	-4	-2	0	2

$$y = x + 1$$

x	-1	0	1	2
y	0	1	2	3

To solve  $2x - 2 = x + 1$ , we need to find the point on both lines where the  $y$ -coordinate is the same.

This is the point where the 2 lines intersect.

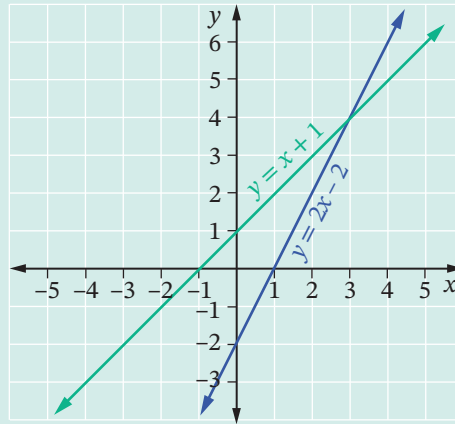
This point is  $(3, 4)$ , which means that the solution to  $2x - 2 = x + 1$  is  $x = 3$ .

Check:

$$\text{LHS} = 2 \times 3 - 2 = 4$$

$$\text{RHS} = 3 + 1 = 4$$

$$\text{LHS} = \text{RHS}$$



Line intersections

**EXERCISE 12.10** ANSWERS ON P. 587

**Intersecting lines UFR**

This exercise is best completed using dynamic geometry software.

- 1** By substitution, find the point of intersection of the lines  $y = 2x - 1$  and  $y = 5 - x$ .  
Select the correct answer **A**, **B**, **C** or **D**. **R**

**A** (2, -1)                      **B** (2, 3)                      **C** (5, -1)                      **D**  $(\frac{4}{3}, \frac{5}{3})$

- 2** Graph each pair of equations and find their point of intersection.

**a**  $y = x$  and  $y = -x + 2$                       **b**  $y = x + 1$  and  $y = 2x$   
**c**  $y = -x - 1$  and  $y = x + 1$                       **d**  $y = \frac{x}{2} - 2$  and  $y = -x + 1$   
**e**  $y = 2x + 1$  and  $y = x + 3$                       **f**  $y = -2x - 1$  and  $y = x + 5$

- 3** What is the point of intersection of  $y = x + 1$  and  $y = -x + 5$ ? Select **A**, **B**, **C** or **D**.

**A** (3, 2)                      **B** (2, 3)                      **C** (4, 1)                      **D** (4, 5)

- 4** Solve each equation graphically. **R**

**a**  $2x - 1 = x + 2$                       **b**  $3x - 2 = 2x - 1$                       **c**  $x - 2 = 4 - x$   
**d**  $2x = x + 3$                       **e**  $3x + 2 = x - 1$                       **f**  $-2x + 3 = 2x + 1$

- 5** Solve each of the equations above algebraically.

- 6** These 3 linear equations form a triangle when graphed on the number plane.

$y = x + 2$                        $y = \frac{x}{2} - 1$                        $y = -3x + 6$

Find the coordinates of the 3 vertices of the triangle. **R**



A page of number planes



Number plane grid paper



EXAMPLE 14

**Power plus** ANSWERS ON P. 589

- 1 a** George receives twice as much as Bill in pocket money. This can be represented by  $y = 2x$ , where  $y$  is George's pocket money and  $x$  is Bill's amount of money. Draw the graph of this line on a number plane where the  $x$ -axis runs from 0 to 12 and the  $y$ -axis from 0 to 24.
- b** The total of George and Bill's pocket money is \$24. Write an equation that represents this.
- c** Draw the graph of your equation in part **b** on the same set of axes as part **a**.
- d** Use your graphs to find George's and Bill's pocket money.

- 2** Graph each equation accurately and state whether it is linear or non-linear.

**a**  $y = x^3 - 2$                       **b**  $x + y = 5$                       **c**  $y = x^2 - 1$   
**d**  $y = 4 - x^2$                       **e**  $x - y = 4$                       **f**  $y = \frac{4}{x}$



# CHAPTER 12 REVIEW

## Language of maths

coefficient	collinear	consecutive	constant term
coordinates	decreasing	formula	graphically
horizontal	increasing	intersection	linear
number plane	origin	quadrant	satisfy
solution	table of values	term	variable
vertical	x-axis	y-axis	y-intercept



Functions and graphs

- 1 What is the difference between **linear** and **collinear**?
- 2 In which quadrant of the number plane are the  $x$ - and  $y$ -coordinates of a point both negative?
- 3 What are **consecutive** numbers?
- 4 What is the 'answer' to an equation called?
- 5 What does it mean if the coordinates of a point **satisfy** the equation of a line?
- 6 For the linear equation  $y = 4x + 1$ , which is the **constant term**?

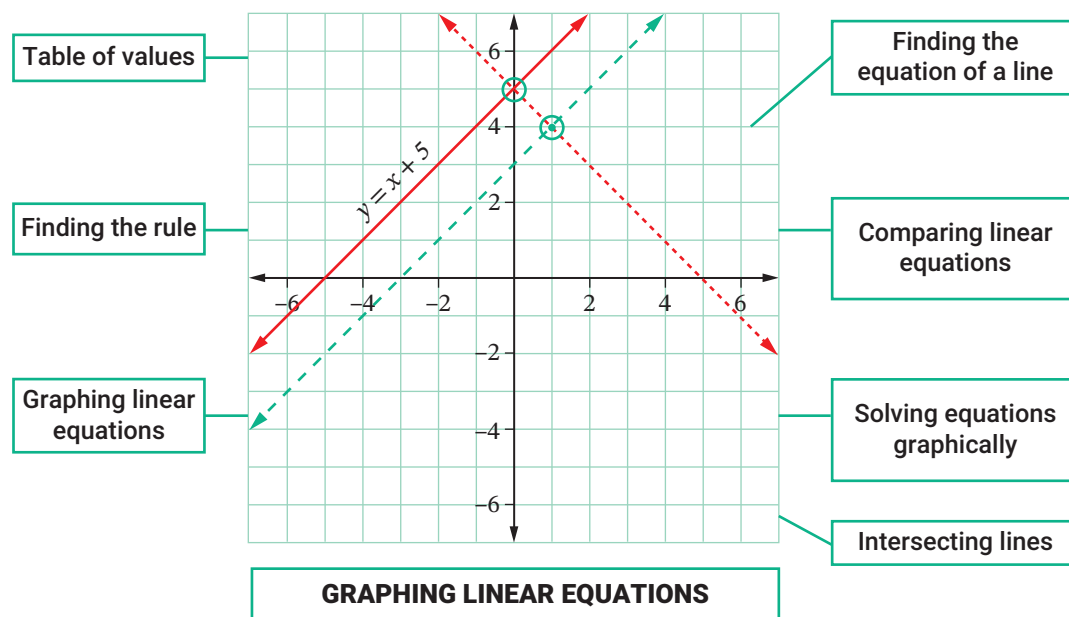
## Topic summary

- Write in your own words what you have learnt about linear equations and their graphs.
- What parts of this topic did you have difficulty with? Discuss them with a friend.
- Where might you use the skills acquired in this chapter?



Mind map:  
Graphing  
linear  
equations

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.





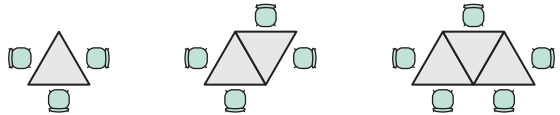
**7** Which point from question 6:

- a** is on the  $y$ -axis?
- b** is in the 2nd quadrant?
- c** is on the  $x$ -axis?
- d** is in the 4th quadrant?

12.04

**8 a** Copy and complete this table for the pattern of tables and chairs.

Number of tables ( $x$ )	1	2	3	4	5
Number of chairs ( $y$ )					



12.05

- b** Find the formula for  $y$ .
- c** Graph the table of values.

**9** Graph each linear equation on a number plane after completing a table of values.

- a**  $y = 2x - 1$
- b**  $y = -x + 3$
- c**  $y = \frac{1}{2}x - 2$

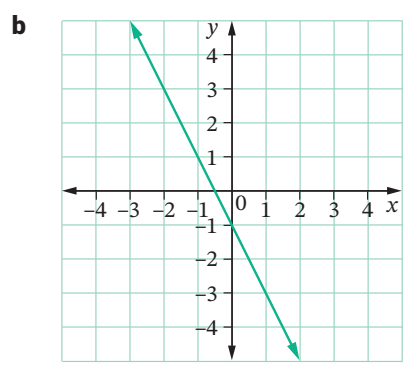
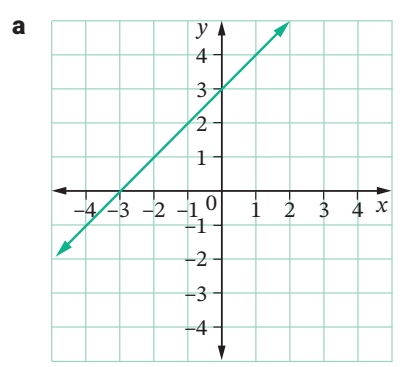
12.06

**10** Test whether each point lies on the line  $y = 3x + 2$ .

- a**  $(1, 2)$
- b**  $(-1, 4)$
- c**  $(-2, -4)$
- d**  $(0, 2)$

12.06

**11** Find the equation of each line.



12.07

**12** Graph each linear equation on the same number plane.

- a**  $y = -3$
- b**  $x = 4$

12.07

**13 a** Graph  $y = -x + 2$ ,  $y = x + 2$  and  $y = 2x + 2$  on the same number plane.

- b** What is the same about the 3 lines?
- c** What is the same about the 3 linear equations?
- d** What is different about the 3 lines?

12.08

**14 a** Graph  $y = 3x - 4$  and use it to solve  $3x - 4 = 5$  graphically.

**b** Graph  $y = 2x + 1$  and use it to solve  $2x + 1 = -2$  graphically.

12.09

**15** Graph  $y = 2x$  and  $y = x - 2$  and find their point of intersection.

12.10

**16** Solve the equation  $3x - 3 = x + 3$ :

- a** graphically
- b** algebraically

12.10